Solving a generic hyperbolic formulation of moment approximations of Boltzmann Equation

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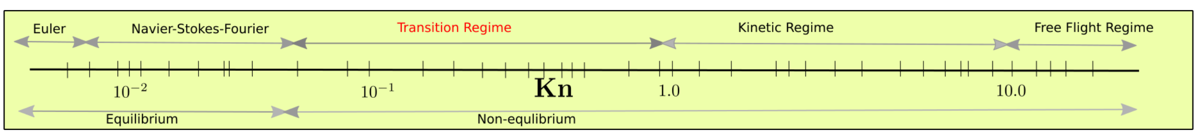
Developing computationally feasible mathematical models for simulating non-equilibrium gas flows accurately remains an active research area. Classical models such as Navier-Stokes and Fourier do not provide statisfactory results in the presence of strong non-equilibrium. On the other end, kinetic models such as Boltzmann equation are accurate but are very expensive to solve numerically. Therefore, it is necessary to develope mathematical models to describe non-equilibrium gas flows accurately and at an acceptable compuational cost.

In kinetic gas theory, Knudsen number (Kn) is a dimensionless quantity which is the ratio of distance travelled by particles between successive collisions and characteristic length of the problem. The following figure illustrates the Knudsen scale.



When the Kn of a process is between 0.1 to 1.0, then it is in the transition regime. We develope models which tackle problems which lie in this regime.

In kinetic theory, moment method is a technique which relates a microscopic description – kinetic model – of a system to a macroscopic description – continuum model – of the system. By following the moment method procedure to the Boltzmann equation, one can derive a set of partial differential equations, which we name moment equations. See [1]. These equations have many desirable properties: *hyperbolicity* which indicates wellposedness of the problem, *symmetry* which comes in handy during calculations, *hierarchical structure* which facilitates generalisation of the problem to arbitrary order, and *rotational invariance* which preserves the structure of the equations during coordinate transformations.

In this work, we propose a generic formulation of moment equations of orbitrary order. By exploiting the above-mentioned propeties of moment equations, we have developed a generic numerical solver which uses finite element method. It is called Fenics For Moment Equations (F2ME). See [2]. This python-based solver is developed based on the existing FEniCS library, a FEM library. The highlights of F2ME are its ability to solve arbitrary order of moment equations and its applicability to arbitrary compuational domains. This work has been validated by comparing with analytical solutions in the case of simplified problem setup and by comparing with results from other well-established commercial softwares, such as ANSYS.

References

[1] Buenger, Christhuraj, Hanke, Torrilhon, Structured Derivation of Moment Equations and Stable Boundary Conditions with an Introduction to Symmetric, Trace-free Tensors, *Kinetic Related Models*, 2022, (submitted)

[2] Christhuraj, *Fenics For Moment Equations,* GitHub, `https://github.com/19ec94/f2me`

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