Non-uniqueness of Leray solutions of the forced Navier-Stokes equations

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In his seminal work, Leray demonstrated the existence of global weak solutions, with nonincreasing energy, to the Navier-Stokes equations in three dimensions. In this talk we exhibit two distinct Leray solutions with zero initial velocity and identical body force.

The starting point of our construction is Vishik's answer to another long-standing problem in fluid dynamics, namely whether the Yudovich uniqueness result for the 2D Euler system can be extended to the class of L^p -integrable vorticity. Building on Vishik's work, we construct a 'background' solution which is unstable for the 3D Navier-Stokes dynamics in similarity variables; the second solution from the same initial datum is a trajectory on the unstable manifold associated to the background solution, in accordance with the predictions of Jia and Sverak.

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