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Positivity and Linear Invariant Preserving Schemes: A new Theorem on Stability

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In recent years, numerical methods that preserve properties such as positivity of the solution of differential equations have gained increasing attention. Applying the so-called Patankar-trick to an explicit Runge–Kutta scheme the positivity of the corresponding Patankar–Runge–Kutta method can be ensured for all time step sizes. However, thereby the schemes become implicit and do not longer preserve other properties of the solution such as conservativity. The class of modified Patankar–Runge–Kutta (MPRK) schemes includes higher-order numerical methods that guarantee the positivity and conservativity of the numerical solution of a positive and conservative production-destruction system irrespectively of the chosen time step size. These schemes are used in many fields due to their high accuracy and robustness. However, by construction they do not belong to the class of general linear methods, so that the application even to linear systems leads to a nonlinear relation between any two iterates.

Moreover, the presence of linear invariants such as conservativity of the underlying continuous problem leads to non-hyperbolic fixed points of the numerical method. Therefore, in order to understand the stability of positivity-preserving schemes with linear invariants, it is crucial to understand the stability of non-hyperbolic fixed points of a nonlinear iteration scheme.

In this talk we present a theorem based on the center manifold theory for maps that allows to study the stability of such non-hyperbolic fixed points of a one-step method. This theorem provides sufficient conditions for the stability of the method as well as the local convergence of the iterates to the steady state of the underlying initial value problem. Additionally, we apply this novel result to investigate the stability properties of, for instance, second order MPRK schemes. Furthermore, numerical experiments are shown which confirm the theoretical results.

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References

[1] Th. Izgin and S. Kopecz and A. Meister. *On Stability of Positive and Conservative Time Integrators and Application to Second Order Modified Patankar–Runge–Kutta Schemes* https://arxiv.org/abs/2202.01099, 2022.

[2] Th. Izgin and S. Kopecz and A. Meister. *On the Stability of Unconditionally Positive and Linear Invariants Preserving Time Integration Schemes* https://doi.org/10.48550/arXiv.2202.11649, 2022.

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