Structured equations in biology; entropy and Monge-Kantorovich distance

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Models arising in biology are often written in terms of Ordinary Differential Equations. The celebrated paper of Kermack-McKendrick (1927), founding mathematical epidemiology, showed the necessity to include parameters in order to describe the state of the individuals, as time elapsed after infection. During the 70s, many mathematical studies were developed when equations are structured by age, size, more generally a physiological trait. The renewal, growth-fragmentation are the more standard equations.

The talk will present structured equations, show that a universal generalized relative entropy structure is available in the linear case, which imposes relaxation to a steady state under non-degeneracy conditions. In the nonlinear cases, it might be that periodic solutions occur, which can be interpreted in biological terms, e.g., as network activity in the neuroscience.

When the equations are conservation laws, a variant of the Monge-Kantorovich distance (called Fortet-Mourier distance) also gives a general non-expansion property of solutions.

Acknowledgements

References

[1] Philippe Michel, Stéphane Mischler, and Benoît Perthame. General relative entropy inequality: an illustration on growth models. *Journal de mathématiques pures et appliquées*, 84(9):1235–1260, 2005.

[2] Nicolas Fournier and Benoît Perthame. A non-expanding transport distance for some structured equations. *SIAM J. Math. Anal.*, 53 (2021), no. 6, 6847-6872.

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