Analysis of models of superfluidity

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This work deals with the rigorous analysis of two models of superfluidity. One of them is a macro-scale description of the interacting dynamics of a mixture of superfluid Helium and normal Helium. The equations used are modifications of the incompressible Navier-Stokes equations in 2D, with a nonlinear *mutual friction* that couples the two fluids. We show global well-posedness of strong solutions (with high-regularity data) to this model, by proving a Beale-Kato-Majda-type condition. This work has been published in the Journal of Nonlinear Science [3].

Next, we study a micro-scale model (the "Pitaevskii model") of superfluid-normal fluid interactions, derived by Lev Pitaevskii in 1959. This involves the nonlinear Schrödinger equation and incompressible inhomogeneous Navier-Stokes equations. Mass and momentum exchange between the two fluids is mediated through a nonlinear and bidirectional coupling. We establish the existence of local solutions (strong in wavefunction and velocity, weak in density) that satisfy an energy equality. The analysis of this model has been published in the Journal of Mathematical Fluid Mechanics [2]

Finally, we prove a weak-strong type uniqueness theorem for the solutions of the Pitaevskii model. The analysis is presented in [1]. This talk is based on joint work with my Ph.D. student Pranava Jayanti.

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References

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