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Tensor PDE model of biological network formation

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We study an elliptic-parabolic system of partial differential equations describing formation of biological network structures. The model takes into consideration the evolution of the permeability tensor under the influence of a diffusion term, representing randomness in the material structure, a decay term describing metabolic cost and a pressure force [1]. A Darcy's law type equation describes the pressure field.

We present a number of numerical examples for the 2D Cai-Hu model, using a semi-implicit solver. The background chosen corresponds to leaf venations according to [2]. The main difficulties are that such models can predict formation of branches and dentritic formation. Since the numerical solution are very sensitive to the numerical parameters, it is very difficult to get a numerical solution that resolves all small scales, unless the problem is sufficiently regularised.

The system of PDE we study consists of a Poisson equation coupled to a reaction-diffusion equation, and the numerical schemes we consider to discretize in space and time follow the strategy we use in [3]. In this paper we construct a second order numerical scheme in time, adapting the traditional ADI method with an extrapolation technique for the pressure. In space we consider a finite difference scheme to discretize the space derivatives, ensuring the second order and the exact conservation of the solution (up to the machine precision).

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References

[1] G. Albi, M. Burger, J. Haskovec, P. Markowich, M. Schlottbom: *Continuum Modelling of Biological Network Formation.* In: N. Bellomo, P. Degond, and E. Tamdor (Eds.), *Active Particles Vol.I - Theory, Models, Applications, Series: Modelling and Simulation in Science and Technology*, Birkhauser-Springer (Boston), 2017.

[2] D. Hu and D. Cai. *Adaptation and optimization of biological transport networks.* Phys. Rev. Lett., 111:138701, Sep 2013.

[3] A. Raudino, A. Grassi, G. Lombardo, G. Russo, C. Astuto, M. Corti, *Anomalous sorption kinetics of self-interacting particles by a spherical trap*, Communications in Computational Physics 31 (2022) 707-738

[4] J. Haskovec, P. Markowich, B. Perthame: *Mathematical Analysis of a PDE System for Biological Network Formation*. Comm. PDE 40:5, pp. 918-956 (2015).

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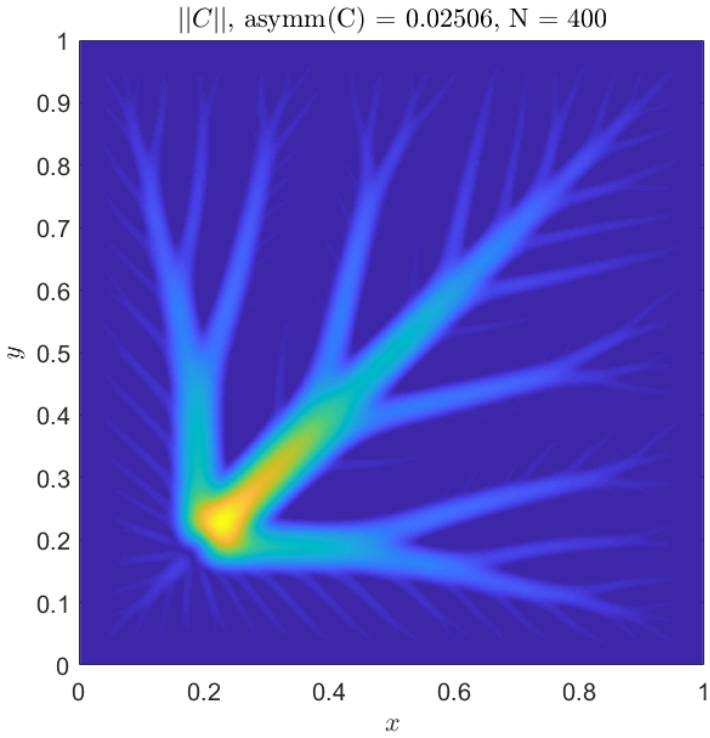

Figure 1: *Typical solution for leaf venation.*