XVIII International Conference on Hyperbolic Problems
Theory, Numerics, Applications
Málaga, June 20-24, 2022

A variational scheme for hyperbolic obstacle-type problems

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In joint works with M. Bonafini, M. Novaga, G. Orlandi, we aim to study a class of hyperbolic obstacle type problems, more precisely considering the following equations:

$$
\begin{cases}
u_{tt} + (-\Delta)^s u + \nabla_u W(u) = 0 & \text{in } (0,T)\times\Omega \\
u(t,x) = 0 & \text{in } [0,T]\times(\mathbb{R}^d\setminus\Omega) \\
u(0,x) = u_0(x) & \text{in } \Omega \\
u_t(0,x) = v_0(x) & \text{in } \Omega
\end{cases}
\tag{1}
$$

for $\Omega \subset \mathbb{R}^d$ an open bounded domain with Lipschitz boundary, $u\colon [0,\infty)\times\mathbb{R}^d \to \mathbb{R}^m$, $m \geq 1$, W a continuous potential with Lipschitz continuous derivative, and for $s > 0$ the operator $(-\Delta)^s$ stands for the fractional s-Laplacian. By using minimizing movements scheme De Giorgi, we provide the existence results for the equation (1) both obstacle-free case (there is no obstalce) and obstacle case (in presence of obstacle g), which include also fractional operators as well as higher dimensional cases, hence extending previous results mostly treated in linear and 1-dimensional case (e.g. [9, 7, 4]). In addition, we discuss some applications to singular limits (with potentials $\frac{1}{\epsilon^2}$ W) of nonlinear wave equations (W is a balance double well potential, $s = 1$ and $m = 1, 2$), which is motivated by the fact that in this case certain solutions of nonlinear wave equations giving rise to interfaces (or defects) evolving by curvature such as minimal surfaces in Minkowski space (e.g. [6, 1, 8]). Finally, we apply our results to study nonlinear waves in adhesive phenomena (e.g. the interacting between strings or membranes with a rigid substrate through an adhesive layer, in this scenario W responds for the energetic contribution of glue layer, and it has less regularity), our results embrace higher dimension and fractional operators extending the previous work initially studied in ([5]). This talk is based on the works [2, 3, 4].

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