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Sustained oscillations in Hyperbolic-Parabolic systems of Viscoelasticity

Athanasios E. TZAVARAS *

The content of this talk is motivated by some questions raised regarding the existence and uniqueness theory for viscoelasticity of Kelvin-Voigt type with non-convex stored energies. Existence theories for the system of viscoelasticity of Kelvin-Voigt type

$$\begin{aligned} \partial_t v - \operatorname{div}(S(F)) &= \Delta\, v \\ \partial_t F - \nabla\, v &= 0 \\ \operatorname{curl}\, F &= 0\,, \end{aligned} \tag{1}$$

with $S(F) = \frac{\partial W(F)}{\partial F}$, are provided (among other places) in References [1] and [3]. The analysis in [3] is based on propagation of H^1-regularity for the deformation gradient of weak solutions in two and three dimensions assuming that the stored energy satisfies the Andrews-Ball condition, in particular allowing for non-monotone stresses. In dimension $d = 2$, weak solutions with deformation gradient in H^1 are in fact unique. On the other hand there is an available existence theory for initial data $v_0 \in L^2$, $F_0 \in L^p$ already available from [1].

This raises the issue what is the difference between the existence theory with F in H^1 and the one in L^p, which is the subject of this talk. This question is already undertaken in [3] where the one dimensional models for phase transitions,

$$\begin{aligned} u_t &= v_x \\ v_t &= \sigma(u)_x + v_{xx}\,, \end{aligned} \tag{2}$$

is examined, and a class of oscillatory solutions are constructed. The examples combine the universal class of uniform shearing solutions with an observation of Hoff [2] that (2) admits solutions with discontinuities in the strain and strain-rate. Combining these ingredients, exact oscillatiory solutions are constructed for (2) with non-monotone stress-strain laws.

This example can be generalized in several space dimensions [4]. For the system (1) a class of oscillatory solutions are constructed that present persistent oscillations. They combine again simple shear deformations jumping across interfaces, analogous to (time dependent) twinning deformations familiar from the theory of phase transitions. It can be shown that such solutions are possible only when the rank-1 convexity condition

$$\frac{\partial^2 W}{\partial F_{i\alpha} \partial F_{j\beta}}(F)\, \xi_i \xi_j \nu_\alpha \nu_\beta > 0 \quad \forall \xi \neq 0\,,\, \nu \in \mathcal{S}^2 \tag{3}$$

is violated. Moreover it is shown that given a class of approximate solutions (v_n, F_n) of (1) bounded in energy and associated with a convex stored energy function $W(F)$, the Young measure associated with sequence of approximate solutions (v_n, F_n) is a Dirac measure if and only if the initial data F_{0n} converge strongly to F_0.

The existence of solutions with persistent oscillations will also be indicated for certain more general systems in two potential directions: (i) For a class of systems in thermoviscoelasticity including the energy equation, and (ii) for multi-dimensional models expressing shearing motions for viscoelastic models with phase transitions. In all these cases the existence of such solutions applies to problems with partly parabolic character but where the first-order part lacks hyperbolicity. The issue of what types of linear hyperbolic-parabolic structures give rise to oscillatory solutions will be discussed.

*Computer, Electrical and Mathematical Science and Engineering Division, King Abdullah University of Science and Technology (KAUST), Thuwal 23955-6900, Saudi Arabia Email: athanasios.tzavaras@kaust.edu.sa

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