XVIII International Conference on Hyperbolic Problems
Theory, Numerics, Applications
Málaga, June 20-24, 2022

Uniqueness of conservative solutions for the Hunter–Saxton equation

Katrin Grunert *

Solutions of the Hunter–Saxton equation

$$u_t(t,x) + uu_x(t,x) = \frac{1}{4}\left(\int_{-\infty}^{x} u_x^2(t,z)dz - \int_{x}^{\infty} u_x^2(t,z)dz\right)$$

might enjoy wave breaking in finite time. This means that even classical solutions, in general, do not exist globally, but only locally in time, since their spatial derivative might become unbounded from below pointwise, while the solution itself remains continuous. Furthermore, energy concentrates on sets of measure zero when wave breaking occurs. Thus the prolongation of solutions beyond wave breaking in the weak sense is non-unique and depends heavily on how the concentrated energy is manipulated. The two most prominent ones are called dissipative, i.e., the concentrated energy is taken out [1] and conservative, i.e., the energy is given back into the system [2]. The existence of conservative solutions has been established with the help of a generalized method of characteristics, which allows to rewrite the Hunter–Saxton equation as a system of ODEs in Lagrangian coordinates.

The aim of this talk is to highlight the main steps in proving the uniqueness of conservative solutions [3]. The overall idea is to establish a bijection between the properties satisfied by each conservative solution and the solution operator from [2]. The main ingredients are measure transport equations, ideas from sub- and supersolutions as well as a good understanding of the characterisation of equivalence classes in Lagrangian coordinates.

This is joint work with H. Holden.

Acknowledgements

This research has been supported by the grants *Waves and Nonlinear Phenomena (WaNP)* and *Wave Phenomena and Stability - a Shocking Combination (WaPheS)* from the Research Council of Norway.

References

[1] A. Bressan and A. Constantin. Global solutions of the Hunter–Saxton equation. *SIAM J. Math. Anal.* 37 (2005) 996–1026.

[2] A. Bressan, H. Holden, and X. Raynaud. Lipschitz metric for the Hunter–Saxton equation. *J. Math. Pures Appl. (9)* 94 (2010) 68–92.

[3] K. Grunert and H. Holden. Uniqueness of conservative solutions for the Hunter–Saxton equation. arXiv:2107.12681.

*NTNU Norwegian University of Science and Technology. Email: katrin.grunert@ntnu.no