XVIII International Conference on Hyperbolic Problems
Theory, Numerics, Applications
Málaga, June 20-24, 2022

Extending the BGK model via entropy minimization

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The fundamental model I am considering is the so called Bathnagar-Gross-Krook (BGK) equation:

$$\partial_t f + v \cdot \nabla_x f = \nu(M(f) - f)$$

where f is the distribution function of the gas, $M(f)$ the equilibrium distribution (in the classical case a Maxwell distribution) and ν describes the collision frequency. The BGK equation is an approximation of the Boltzmann equation which has the same main properties as Boltzmann (conservation of properties, H-Theorem and the same shape of the equilibrium) and is much more efficient in numerical simulations. Therefore, in many applications, this approximation is used. However, the standard BGK equation is too simple in real applications. Therefore a recent open question is if it is possible to equip this approximation with more physics to describe more complex phenomena but still try to keep the numerical efficiency.

In this talk, I will focus on two examples of extending the BGK model via minimization of a corresponding entropy function. The first example deals with including a velocity dependency into the collision frequency and the second example with including quantum effects. In both cases the Maxwell distribution has to be exchanged by a different function which leads to difficulties in the proof of the conservation properties and the H-Theorem.

In the first case, if we replace the collision frequency by a velocity dependent collision frequency, one has to replace the Maxwell distribution by a different function such that the conservation properties are still true. I will show in the talk that such a function exists (in one species and in gas mixture case) and if we assume a certain shape which is motivation by an entropy minimization that this choice is unique.

In the second part of the talk, I will talk about extensions to quantum gas mixtures. In this case, one replaces the Maxwell distribution by a Fermi of a Bose distribution. Also in this case, it is possible to prove that there exists Fermi or Bose distributions in the case of gas mixtures such that we still have conservation properties and an H-Theorem.

Finally, I will give an outlook for extending this strategy to cover a large class of BGK models including for example the quantum BGK model with velocity dependent collision frequency, the relativistic BGK model with velocity dependent collision frequency, BGK models for gas mixtures and BGK models for polyatomic molecules.

This is joint work with S. Yun, G. Bae, (Sungkyunkwan University), J. Haack (Los Alamos National Laboratory), C. Hauck (Oak Ridge National Laboratory) and C. Klingenberg, S. Warnecke (University of Wuerzburg).

Acknowledgements

This research has been partially funded by the Alexander von Humboldt foundation and the DfG project with grant number 456849348.

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