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Uncertainty quantification in hierarchical vehicular flow models

Michael Herty [*], Elisa Iacomini [†]

Although traffic models have been extensively studied, obtaining trustful forecast from these models is still challenging, since the evolution of traffic is also exposed to the presence of various sources and types of uncertainties. Indeed uncertainty may stem from real data affected by errors in the measurements or variations in the behavior of vehicular traffic.
In this talk, we will investigate the propagation of uncertainties in traffic flow models [3], especially how the input uncertainty propagates through the models.

First, we consider kinetic vehicular traffic flow models of BGK type [4]. Considering different spatial and temporal scales, those models allow to derive a hierarchy of traffic models including a hydrodynamic description.
The kinetic BGK–model is extended by introducing a parametric stochastic variable to describe possible uncertainty in traffic. The interplay of uncertainty with the given model hierarchy, shown in the deterministic case in [4], is studied in detail. Theoretical results on consistent formulations of the stochastic differential equations on the hydrodynamic level are given. Indeed, we study in details the stochastic Aw-Rascle-Zhang model [2], which belongs to a class of second order macroscopic traffic flow models described by a system of nonlinear hyperbolic equations.

The treatment of stochastic models can be either non–intrusive, e.g., based on sampling (Monte–Carlo) or based on collocation, or intrusive. Here, we focus on the latter approach since it allows us to recover stochastic quantities of interest at each point in space and time directly from the formulation of the problem. Stochastic input is represented by a series of orthogonal functions, known as generalized polynomial chaos (gPC) expansions [5, 6, 7]. Substituting the expansions in the evolution equations and applying the Galerkin projection lead to a deterministic system for the coefficients of the truncated series, due to the orthogonality of the basis functions. We follow this intrusive approach in order to investigate how uncertainty propagates between the kinetic and the fluid flow hierarchy of description. The established links are depicted in Figure 1.

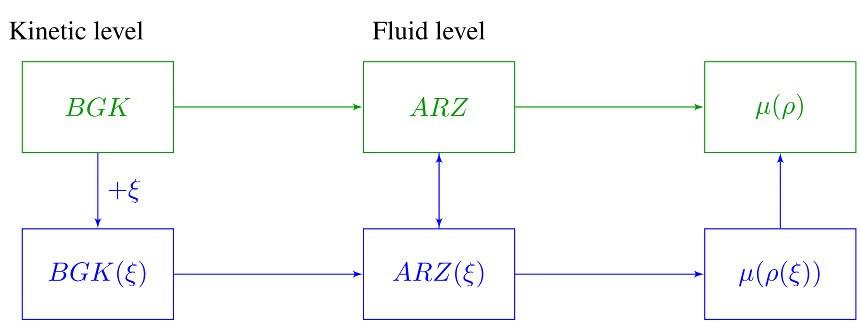

Figure 1: Outline of the model hierarchy. The left two columns indicate the kinetic and fluid description of traffic flow as presented in [4]. The third column refers to the diffusion coefficient $\mu(\rho)$ to classify traffic instabilities. The green hierarchy is deterministic while the blue includes a parametric uncertainty ξ. The indicated links are established.

However, results for nonlinear hyperbolic systems, i.e. for systems at the fluid level, are only partial, since desired properties like

[*]Institut für Geometrie und Praktische Mathematik, RWTH Aachen University, Templergraben 55, 52062 Aachen, Germany Email: herty@igpm.rwth-aachen.de
[†]Institut für Geometrie und Praktische Mathematik, RWTH Aachen University, Templergraben 55, 52062 Aachen, Germany. Email: iacomini@igpm.rwth-aachen.de

hyperbolicity are not necessarily transferred to the intrusive formulation [1]. We show that under suitable assumptions, hyperbolicity is recovered for the Aw-Rascle-Zhang and accordingly to the deterministic case, conservative form and non-conservative form are proved to be equivalent for smooth solutions also in the stochastic Galerkin formulations.

Moreover, from the kinetic description and the BGK–type models, we perform a Chapman–Enskog expansion to obtain the diffusion coefficient $\mu(\rho)$ which has been used to classify possible unstable traffic regimes in the deterministic case. Here we translate the the procedure on the stochastic level, which allows to characterize possible traffic zones of high risk.

Several numerical tests will illustrate the theoretical results. The numerical convergence of the truncated expansion is shown both in case of a rarefaction and a shock wave. Furthermore, the effect of the possibly negative diffusion in the stochastic hydrodynamic model is numerically investigated. Indeed, we are interested in indicating and forecast regions of high risk of congestion or instabilities. To this aim, we identify the parameters influencing regions of high probability of instabilities, then we show the evolution in time of the probability of possibly negative diffusion coefficient.

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