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A very easy high-order well-balanced reconstruction for hyperbolic systems with source terms

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Our goal is to apply high-order finite volume schemes based on MUSCL reconstruction techniques to approximate the weak solutions of hyperbolic systems with source terms. More specifically, we are interested in so-called well-balanced schemes, which are able to exactly preserve the steady state solutions of such systems. In this context, the preservation of such solutions turns out to be very challenging. Indeed, the designed reconstruction must preserve the equilibria under consideration in order to satisfy the required well-balanced property. A priori, to capture such steady solutions, one needs to solve some strongly nonlinear equations.

In [1], we designed a very easy correction to high-order finite volume methods. Given a first-order well-balanced scheme, this technique ensures the well-balanced property for any finite volume scheme of order greater than or equal to 2, e.g. MUSCL-type schemes. This correction ensures that such schemes exactly preserve steady solutions. The main discrepancy with usual techniques lies in avoiding the inversion of the nonlinear function that governs the steady solutions. Moreover, some systems, such as the Euler equations with gravity [2], involve under-determined equilibria. In such cases, additional assumptions on the unknowns must be introduced. This leads to several nonlinear functions, corresponding to different classes of steady state equations, having to be considered separately. An example of such behavior are the polytropic and isothermal equilibria of the Euler equations with gravity. Since the derived correction only considers the evaluation of the nonlinear functions governing the steady states, rather than solving the corresponding nonlinear equilibrium systems including additional closure assumptions, we are able to deal with under-determined stationary systems.

In this communication, the correction framework is first presented for a second-order scheme, and then applied to the shallow water and Euler equations to verify the scheme's properties in several numerical experiments.

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References

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