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Convergence of FE Based Schemes for the Euler Equations via Dissipative Weak Solutions

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Since the Cauchy problem for the complete Euler system is in general ill-posed in the class of admissible entropy weak solutions, the question of convergence analysis of numerical schemes is of fundamental importance. In this context, the concept of dissipative weak solutions seems to be quite a promising approach to analyze this system analytically and numerically. In [1], the authors have studied the convergence of a class of entropy dissipative finite volume schemes for the barotropic and complete compressible Euler equations in the multidimensional case and proved suitable stability and consistency properties to ensure convergence of their FV schemes to a weak dissipative solution. The theory has been further developed for several (classical) FV schemes (of maximum order two) and have been tested numerically, cf. [2, 3].
In this talk, we focus on high-order finite element based methods, explain the basic framework of dissipative weak solutions, their numerical treatment and prove convergence to a dissipative weak solution in multidimensional case. To this end, it is crucial that structure preserving properties, such as positivity preservation and entropy inequality hold and the schemes are consistent with the underlying PDE. We show how to ensure them and demonstrate convergence results of various multidimensional high-order FE based schemes. In numerical simulations, we verify our theoretical findings, cf. Figure 1.

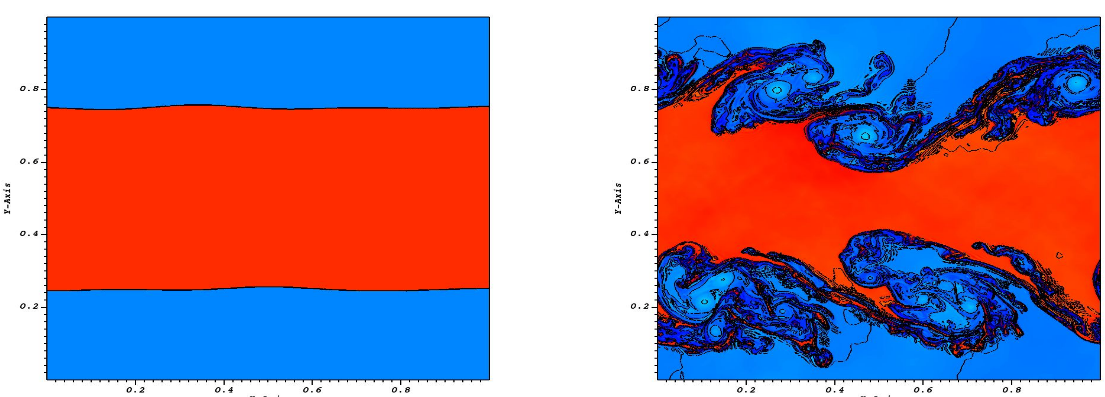

Figure 1: Kelvin-Helmholz instability.

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