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Convergence analysis of some uncertainty quantification methods for compressible Navier-Stokes equations

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In this talk we will present our recent results on rigorous convergence analysis of a collocation finite volume method and the Monte Carlo finite volume method. These uncertainty quantification methods are applied to the random Navier-Stokes equations with uncertain initial data and viscosity coefficients.

Typical convergence analysis of these uncertainty quantification methods available in the literature leans on uniqueness and continuous dependence of solutions on random parameters for the underlying partial differential equations. However, in our case the principal difficulty is due to the fact that the compressible Navier–Stokes equations are not known to be solvable in the class of smooth solutions on a possibly large time interval $(0, T)$ even for smooth initial data. The weak solutions exist globally in time for the adiabatic constant $\gamma > \frac{d}{2}$, see Feireisl [2] and Lions [7]. Their uniqueness in terms of the initial data remains an open problem. Consequently, we are analysing random Navier-Stokes system with low regularity in the stochastic space. For the collocation method we therefore opted for piecewise constant interpolations in the stochastic space. Approximation in space-time is realized by a finite volume method (FVM) with a piecewise constant spatial approximation and upwind numerical fluxes.

In our previous works [3], [5] the convergence and error analysis of the upwind FVM have been done for (deterministic) compressible Navier-Stokes equations. Indeed, for finite energy initial data the numerical solutions converge only weakly* to a dissipative measure–valued solution. However, if the numerical solutions are uniformly bounded, we have strong convergence to the strong solution of the Navier–Stokes system in the corresponding L^p-spaces, $p \geq 1$. Deterministic error estimates are obtained using the relative energy method.

Under a hypothesis that the numerical solutions obtained by our collocation FVM or Monte Carlo FVM are bounded in probability, we will show that they converge strongly in space-time and in probability to an exact random solution, cf. [4]. Our analysis implies that the latter is the global (in time) random strong solution of the Navier-Stokes system. For the convergence/error analysis of the random problem we combine the deterministic convergence/error results with the probabilistic compactness arguments. The latter are based on the application of the Skorohod/Jakubowski theorem [1] and the hypothesis on the boundedness in probability to pass to a limit $\mathbb{P}$-a.s. on a new probability space, where all quantities in question share the same law with the original ones. Finally, we use the Gyöngy–Krylov criterion [6] to show the convergence in probability of the approximate solutions on the original probability space. As a result we derive the convergence/error estimates in probability. For the Monte Carlo FVM this result can be combined with the statistical error estimate of the Monte Carlo estimators to derive the error estimates for the statistical moments. Extensive numerical simulations will illustrate theoretical results.

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