XVIII INTERNATIONAL CONFERENCE ON HYPERBOLIC PROBLEMS
THEORY, NUMERICS, APPLICATIONS
Málaga, June 20-24, 2022

# High-friction limit of the bipolar Euler-Poisson system

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In this talk it will be explained how to obtain the bipolar drift-diffusion system

$$\begin{cases} \rho_t = \nabla \cdot \left(\rho \nabla \dfrac{\delta \mathcal{E}}{\delta \rho}\right) \\ n_t = \nabla \cdot \left(n \nabla \dfrac{\delta \mathcal{E}}{\delta n}\right) \\ -\Delta\phi = \rho - n \end{cases} \quad \text{in } ]0,T[\times\Omega \tag{1}$$

$$\rho \nabla \frac{\delta \mathcal{E}}{\delta \rho} \cdot \nu = n \nabla \frac{\delta \mathcal{E}}{\delta n} \cdot \nu = \frac{\partial \phi}{\partial \nu} = 0 \quad \text{on } [0,T[\times\partial\Omega$$

as the high-friction limit of the bipolar Euler-Poisson system:

$$\begin{cases} \rho_t + \nabla \cdot (\rho u) = 0 \\ (\rho u)_t + \nabla \cdot (\rho u \otimes u) = -\dfrac{1}{\varepsilon}\rho \nabla \dfrac{\delta \mathcal{E}}{\delta \rho} - \dfrac{1}{\varepsilon}\rho u \\ n_t + \nabla \cdot (n v) = 0 \\ (n v)_t + \nabla \cdot (n v \otimes v) = -\dfrac{1}{\varepsilon} n \nabla \dfrac{\delta \mathcal{E}}{\delta n} - \dfrac{1}{\varepsilon} n v \\ -\Delta\phi = \rho - n \end{cases} \quad \text{in } ]0,T[\times\Omega \tag{2}$$

$$u \cdot \nu = v \cdot \nu = \frac{\partial \phi}{\partial \nu} = 0 \quad \text{on } [0,T[\times\partial\Omega,$$

where the functional $\mathcal{E}$ is given by

$$\mathcal{E}(\rho, n) := \int_\Omega h_1(\rho) + h_2(n) + \tfrac{1}{2}|\nabla\phi|^2 dx.$$

The models (1) and (2) describe two charged fluid systems interacting through an electrostatic potential, and are basic models for applications in semiconductor devices or plasma physics. The objective is to describe the relation between the two models, thus extending the framework of convergence for a single fluid system. In order to perform this limiting process, one uses the relative energy method for dissipative weak solutions of (2) and strong and bounded away from vacuum solutions of (1) in several space dimensions. This method provides an efficient mathematical mechanism for stability analysis and establishing limiting processes. Such approach was successful for the relaxation limit in single-species fluid models, as well as for certain (weakly coupled through friction) multicomponent systems.

For this analysis, the first step is to obtain a relative energy inequality satisfied by the relative energy functional. This functional serves as a yardstick for the comparison between the solutions of the two systems. To do so, one first regards system (2) as an approximation of system (1), and then considering the weak formulation for the difference between the two, one chooses test functions that are built up from the strong solution of system (1). After long but straighforward computations one reaches the desired inequality which has on the left-hand-side the relative energy functional of system (2) and on the right-hand-side some error terms. The technical part amounts to bound those error terms with the relative energy functional. The term requiring attention is the one associated with the electric field. Due to the antisymmetry of the electric charges and the fact that the velocities of

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the fluids are distinct, one cannot reproduce the usual arguments to simplify this electric field term. The desired bound is reached using results on Riesz potentials [2] and Neumann functions [1]. This is made possible given that the solution of the Poisson equation is expressed as

$$\phi(t,x) = \big(N * (\rho - n)\big)(t,x) := \int_\Omega N(x,y)\big(\rho(t,y) - n(t,y)\big)dy,$$

and its spatial gradient is understood as

$$\nabla\phi(t,x) = \big(\nabla_x N * (\rho - n)\big)(t,x) := \int_\Omega \nabla_x N(x,y)\big(\rho(t,y) - n(t,y)\big)dy,$$

where $N$ is the Neumann function for the smooth bounded domain $\Omega$.

A Gronwall inequality then yields the relaxation convergence as a stability result, which shows that if a strong solution of (1) is bounded away from vacuum and the initial data converge at the initial time then this convergence is preserved for all times $t \in [0, T[$.

This is a joint work with A. E. Tzavaras.

## References


[1] C. E. Kenig, *Harmonic Analysis Techniques for Second Order Elliptic Boundary Vlue Problems*, Vol. 83, American Mathematical Society, 1994.

[2] E. M. Stein, *Singular Integrals and Differentiability Properties of Functions*, Vol. 2, Princeton University Press, 1970.