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Global BV Solution to a System of Balance Laws from Traffic Flow

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We apply Constantine Dafermos' *Hyperbolic conservation laws in continuum physics* [6] to the higher order continuum model

$$\begin{aligned} &\rho_t + (\rho v)_x = 0, \\ &v_t + (\frac{1}{2}v^2 + g(\rho))_x + \frac{v - v_e(\rho)}{\tau} = 0 \end{aligned} \tag{1}$$

where $x \in \mathbb{R}, t > 0$, ρ is the density, v is the velocity, and $v_e(\rho)$ is the equilibrium velocity. In addition, the anticipation factor g satisfies

$$g'(\rho) = \rho(v_e'(\rho)/\theta)^2, \tag{2}$$

where $0 < \theta < 1$.

We study the global existence of BV solutions to (1) in the framework of Dafermos [6]. We will transform (1) into an equivalent form and apply theory from [6]. Our goal is to verify conditions needed to derive decay of L^1- and L^2- norm with respect to the time variable, admissible BV solution to the Cauchy problem, and admissible BV solution to the Cauchy problem with periodic initial data for (1). In particular, we will derive the entropy-entropy flux pair, Kawashima condition, sub-characteristic condition, and the partial dissipative inequality.

Constructing global solutions and finding zero relaxation limits of traffic flow models have been a recent focus of study [17, 19, 21]. A traffic system can exhibit complicated behavior since it is based on the interactions of roadways, vehicles, and the drivers involved. Factors that need to be considered in analyzing such a system include nonlinear dynamics and human behavior. Both *microscopic* (single vehicles) [27] and *macroscopic* (collective traffic) models [13] have been utilized to deal with this phenomenon. This study is concerned with macroscopic models.

In this work, we take the equilibrium velocity as

$$v_e(\rho) = -a\rho + b, \tag{3}$$

where $a > 0$, $b > 0$. Thus the fundamental diagram $q(\rho) = \rho(-a\rho + b)$, is concave. (3) is an actual equilibrium velocity observed in traffic flow [11] [19].

In our model (1), $0 < \theta < 1$ in (2), whereas $\theta = 1$ in Zhang's higher continuum model [31]. Zhang adopted a relative wave propagating speed to the car speed at equilibrium [19]. We adopt a larger relative speed and obtain the anticipation factor satisfying (2). The anticipation factor expresses the drivers' car following behavior. Larger relative speed implies quicker reaction time, which leads to safer and smoother traffic conditions on highways.

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