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A hybrid kinetic model for inert gas mixtures

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The evolution of a gas in the phase space is classically described by the Boltzmann equation and the contribution of interactions is modeled by an integral nonlinear operator that takes account of the dinamycis of each collision. Unfortunately this approach requires a high computational cost in simulations, especially when one considers a gas mixture and hence a larger number of collision operators (one for each type of interaction) [1].
For such reason, alternative formulations have been proposed since the pioneering model for a single gas proposed by Bhatnagar, Gross and Krook [2], whose idea was to replace the integral nonlinear operator by a simpler linear one reproducing the relaxation of the system towards a Maxwellian state and recovering the usual conservations properties. The extension of this approach to gas mixtures is not unique; a first mathematically rigorous and consistent extension has been proposed in 2002 [3], by introducing a unique global linear relaxation operator instead of the Boltzmann ones, taking into account the effects of collisions with all other species; however, significant information on binary exchanges is lost and recently a new consistent BGK model has been proposed [4], by replacing the sum of Boltzmann terms by a sum of relaxation operators, one for each couple of components.
In this talk we want to propose a new hybrid kinetic model for gas dynamics that combines the positive features of Boltzmann and BGK descriptions, preserving the accuracy of Boltzmann description for a part of the collisional phenomenon and using the BGK approach for the remaining processes to reduce the computational cost. Such approach is very useful to describe proper hydrodynamic regimes, like the one dominated by intraspecies collisions, that is typical when the components of the mixture have very disparate masses (e.g. ions and electrons) [5].
The consistency of the model is proved, focusing in particular on the preservation of global momentum and total energy, positivity of species temperatures and convergence to a global (Maxwellian) equilibrium; moreover, the existence of a Lyapunov functional miming the entropy dissipation is guaranteeed.
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