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Degenerating triangular convection-diffusion systems modelling froth flotation

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Froth flotation is a widely used unit operation, for instance for the recovery of valuable minerals from low-grade ores. The flotation process selectively separates hydrophobic materials (that are repelled by water) from hydrophilic (that would be attracted to water), where both are suspended in a viscous fluid. The process is often applied in a column to which both a mixture of particles (or droplets) and air bubbles are injected. The effluent at the top should consist of a concentrate of hydrophobic particles that are attached to the bubbles, while the hydrophilic particles settle to the bottom, where they are removed. The desired product is the concentrate that is removed through a launder, see Figure 1 (a). The liquid content of the froth is variable and subject to the effect of drainage. The hydrophilic particles (slimes or gangue) do not attach to bubbles, but usually settle to the bottom and are removed continuously. Close to the top, additional wash water can be injected to assist with the rejection of entrained impurities and increase the froth stability. Mathematical models are required for the design, simulation, and control of flotation columns (see [1] for a recent review from a technological viewpoint.)

It is the purpose of this contribution to present advances in the formulation, analysis and numerical solution of a series of one-dimensional models [2–6] describing flotation columns. The ultimate version leads to degenerating, nonlinear convection-diffusion systems with discontinuous flux, where the unknowns are the concentrations ϕ of bubbles and ψ of solids as functions of height z and time t. This system can be written as

$$A(z)\partial_t \begin{pmatrix}\phi\\ \psi\end{pmatrix} + \partial_z\left(A(z)\begin{pmatrix}J(\phi,z,t)\\ -\tilde{F}(\psi,\phi,z,t)\end{pmatrix}\right) = \partial_z\left(A(z)\gamma(z)\partial_z D(\phi)\begin{pmatrix}1\\ -\psi/(1-\phi)\end{pmatrix}\right) + Q_{\mathrm{F}}(t)\begin{pmatrix}\phi_{\mathrm{F}}(t)\\ \psi_{\mathrm{F}}(t)\end{pmatrix}\delta(z-z_{\mathrm{F}}). \tag{1}$$

Here $A(z)$ the cross-sectional area of the tank, and J and $\tilde{F}$ are convective flux functions that depend discontinuously on z at the locations of the feed (z_{F}), the wash water inlet (z_{W}), and the outlets at the top (z_{E}) and at the bottom (z_{U}). The system (1) is valid for $t > 0$ and all $z \in \mathbb{R}$ where the characteristic function $\gamma(z) = 1$ indicates the interior of the tank and $\gamma(z) = 0$ outside, and δ is the delta function. The quantity Q_{F} is the feed volume rate and $\phi_{\mathrm{F}}(t)$ and $\psi_{\mathrm{F}}(t)$ are the feed bubble and (hydrophilic) solids volume fractions, respectively. Outside the tank, the mixture is assumed to follow the outlet streams; consequently, boundary conditions are not needed; conservation of mass determines the outlet volume fractions in a natural way. The nonlinear function D models the capillarity present when bubbles are in contact. Precisely, we define either $D \equiv 0$ (when capillarity is not taken into account, as in [2–5]) or $D(\phi) := \int_0^\phi d(s)\,\mathrm{d}s$. The function d is assumed to satisfy $d(\phi) = D'(\phi) = 0$ for $0 \le \phi \le \phi_{\mathrm{c}}$ and $d(\phi) = D'(\phi) > 0$ for $\phi_{\mathrm{c}} < \phi \le 1$, where ϕ_{c} is a critical bubble volume fraction (that marks the onset of capillarity; here we choose $\phi_{\mathrm{c}} = 0.74$.) Consequently, wherever $\phi \le \phi_{\mathrm{c}}$, (1) degenerates into a first-order system of conservation laws of hyperbolic type. Without going into details, we mention that both J and $\tilde{F}$ depend non-linearly on their ϕ- and ψ-arguments.

The sole ϕ-equation in (1) is a strongly degenerate parabolic equation with discontinuous flux whose well-posedness and numerical analysis can be handled by known arguments (cf. the analysis of a similar model of continuous solid-liquid sedimentation [7]). It is also possible to characterize the conditions under which the system (1) admits stationary solutions, which are of significant practical interest. The parameter space (in terms of feed and outlet flows) admitting such "steady-state" solutions, which are stationary entropy solutions and that are of practical interest, can be conveniently visualized in so-called "operating charts." The analysis of the coupled system (1), however, poses some new challenges. Based on known arguments for certain monotone schemes [8, 9] it has been possible to define a difference scheme for (1) whose solutions satisfy the natural bounds $0 \le \phi \le 1$

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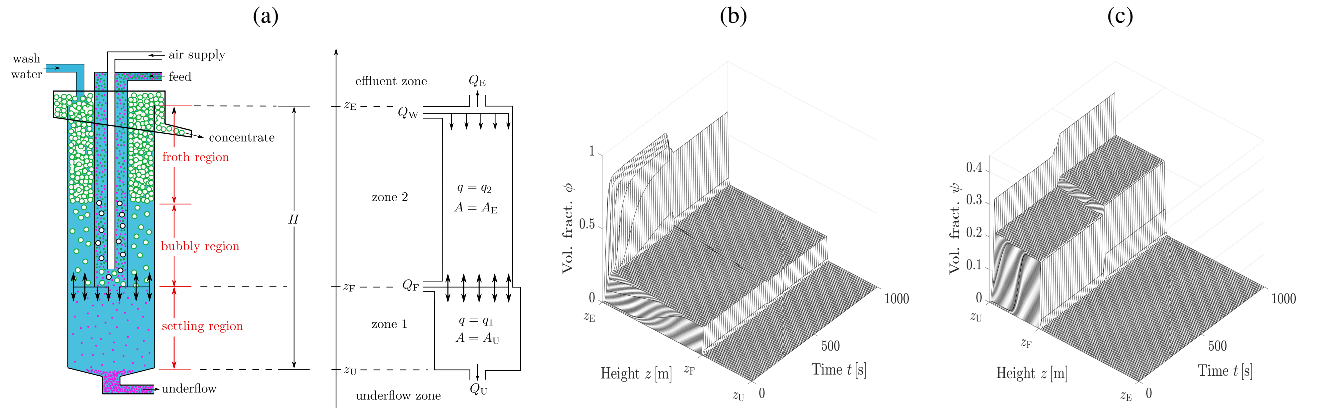

Figure 1: (a) Schematic of a flotation column (left) and one-dimensional conceptual model (right) with a non-constant cross-sectional area $A(z)$. Wash water is sprinkled at the effluent level $z = z_{\mathrm{E}}$ and a mixture of aggregates (bubbles) and feed slurry is fed at $z = z_{\mathrm{F}}$, where $z_{\mathrm{U}} < z_{\mathrm{F}} < z_{\mathrm{E}}$ divide the z-axis into various zones. (b, c) Numerical simulation of (b) the gas volume fraction ϕ, (c) the solids volume fraction ψ during transition between steady states [6].

and $0 \leq \psi \leq 1 - \phi$ at a discrete level [6] (under a suitable CFL condition). Work under preparation addresses the remaining convergence analysis, which starts from the simplified case $\gamma \equiv 1$, $A \equiv \mathrm{const.}$, $D \equiv 0$ (among other simplifications), and invokes compensated compactness arguments known from the analysis of schemes for triangular systems of conservation laws [10, 11].

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References

[1] P. Quintanilla, S.J. Neethling, and P.R. Brito-Parada. Modelling for froth flotation control: A review. *Minerals Eng.*, 162:106718, 2021.

[2] R. Bürger, S. Diehl, and M.C. Martí. A conservation law with multiply discontinuous flux modelling a flotation column. *Netw. Heterog. Media*, 134:339–371, 2018.

[3] R. Bürger, S. Diehl, and M.C. Martí. A system of conservation laws with discontinuous flux modelling flotation with sedimentation. *IMA J. Appl. Math.*, 84:930–973, 2019.

[4] R. Bürger, S. Diehl, M.C. Martí, and Y. Vásquez. Flotation with sedimentation: steady states and numerical simulation of transient operation. *Minerals Eng.*, 157:106419, 2020.

[5] R. Bürger, S. Diehl, M.C. Martí, and Y. Vásquez. Simulation and control of dissolved air flotation and column froth flotation with simultaneous sedimentation. *Water Sci. Tech.* 81:1723–1732, 2020.

[6] R. Bürger, S. Diehl, M.C. Martí, and Y. Vásquez. A degenerating convection-diffusion system modelling froth flotation with drainage. Preprint, Centro de Investigación en Ingeniería Matemática, Universidad de Concepción; submitted.

[7] R. Bürger, K.H. Karlsen, and J.D. Towers. A model of continuous sedimentation of flocculated suspensions in clarifier-thickener units. *SIAM J. Appl. Math.*, 65:882–940, 2005.

[8] B. Engquist and S. Osher. One-sided difference approximations for nonlinear conservation laws. *Math. Comp.*, 36(154):321–351, 1981.

[9] R. Bürger, A. García, K.H. Karlsen, and J.D. Towers. A family of schemes for kinematic flows with discontinuous flux. *J. Eng. Math.*, 60:387–425, 2008.

[10] G.M. Coclite, S. Mishra, and N.H. Risebro. Convergence of an Engquist-Osher scheme for a multi-dimensional triangular system of conservation laws. *Math. Comp.*, 79(269):71–94, 2010.

[11] K.H. Karlsen, S. Mishra, and N.H. Risebro. Convergence of finite volume schemes for triangular systems of conservation laws. *Numer. Math.*, 111(4):559–589, 2008.