XVIII International Conference on Hyperbolic Problems
Theory, Numerics, Applications
Málaga, June 20-24, 2022

Entropy stable and positivity preserving innovative Godunov-type schemes for multidimensional hyperbolic systems of conservation laws on unstructured grid — Bridging Lagrangian and Eulerian frameworks

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In this paper, we propose to reuse the notion of simple Riemann solver in Lagrangian coordinates following Gallice [5] to develop a new Eulerian Finite Volume (FV) scheme in the multi-dimensional case on unstructured meshes [1, 2, 3]. In [2], as a proof of concept we entirely derive the associated first-order accurate cell-centered Eulerian scheme for compressible flows using the Lagrangian to Eulerian correspondence. First, the Lagrangian simple Riemann solver [5] is used as a building block to construct its Eulerian counter-part. This solver inherits by construction the properties of the Lagrangian one, mainly: positivity preservation, entropy dissipation, well-defined CFL condition and wave-speed ordering. From this Riemann solver, a classical two-point first-order Finite Volume Eulerian scheme can be deduced for which the numerical fluxes of a given cell are computed only with respect to two neighbors through a common face. Next, we introduce another Eulerian numerical scheme which involves a multi-dimensional Lagrangian nodal solver [4] leading to the so-called multi-point Riemann solver that involves all surrounding cells, including corner cells. The conservation is no more relying on a one-to-one flux cancellation across a face like for most FV approach. Conversely, in this work conservation is retrieved on a node basis. An associated first-order Eulerian scheme is derived on the basis of this multi-point nodal-based Riemann solver. We prove that this FV multi-point scheme still inherits some good properties (entropic, positive) with the extra-property of coupling all neighbor cells in a consistent way. A set of numerical results on general 2D unstructured grids are presented on several classical two-dimensional test cases, showing that the two-point scheme generates spurious instabilities such as the infamous carbuncle phenomena, figure 1, while the multi-point scheme seems insusceptible to those [7, 2]. Classical and demanding test case results, figure 1, such as Sedov, Noh, Forward Facing step, vortex, odd-even decoupling, etc. will be presented and commented.

If time permits some extensions of this family of Eulerian FV numerical schemes will also be presented, namely the second-order in space/time extension, the 3D and parallel version of these schemes. Also an implicit version has been designed [3] to allow the capture of steady-state solution of hypersonic reentry simulation, see figure 2. At last the treatment of balanced laws has also been studied by solving the Shallow-Water toy model in 2D, [3]. Numerical tests for such extensions will be systematically presented to illustrate the good behavior of this family of Eulerian FV schemes.

References

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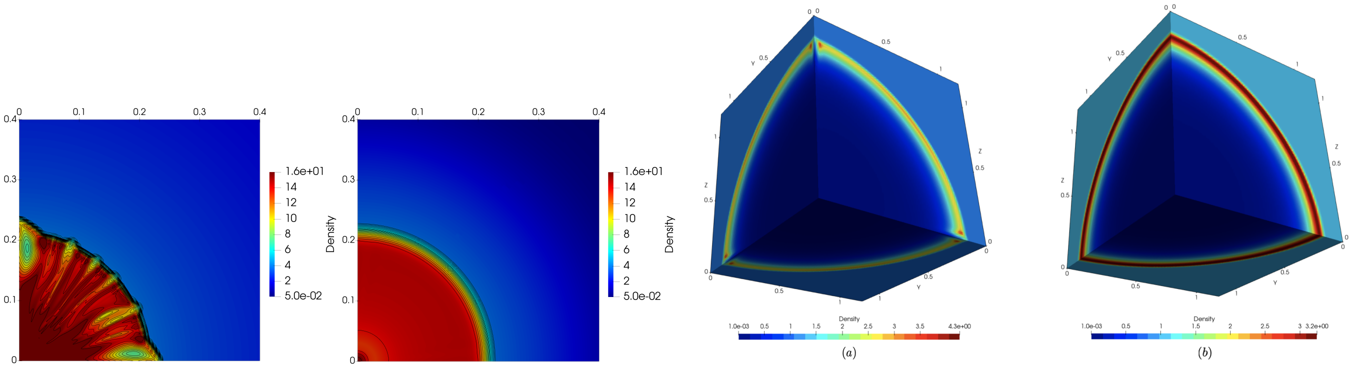

Figure 1: Noh and Sedov problems solved by a classical two-point FV scheme (showing some spurious instability) and the proposed multi-point scheme.

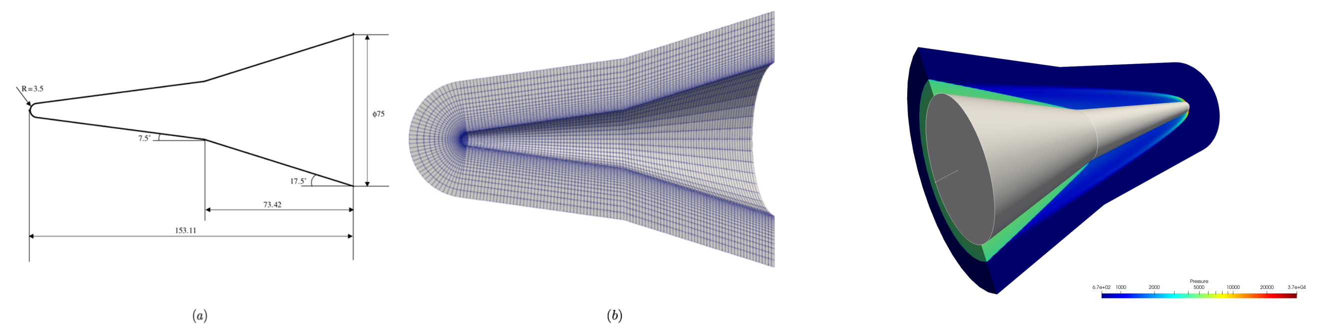

Figure 2: 3D cone-flare simulation. Left: geometry and mesh. Right: pressure.

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