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Compensated Integrability and Conservation Laws

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Compensated Integrability is a recent tool from Functional Analysis. It applies to positive semi-definite tensors whose row-wise Divergence is a finite measure [2, 3]. Quite often, this Divergence vanishes identically. We shall explain why Div-free tensors occur naturally in various models of Mathematical Physics, as a consequence of Nœther's Theorem [4].

Somehow, Compensated Integrability dual to Brenier's existence result for the "second BVP" for the Monge–Ampère equation. It extends in a non-trivial manner classical statements, such as Gagliardo–Nirenberg–Sobolev Inequality, or the Isoperimetric Inequality. In the periodic situation, it expresses the Div-quasiconcavity of $A \mapsto (\det A)^{\frac{1}{n-1}}$ (recall that $A \mapsto (\det A)^{\frac{1}{n}}$ is concave over $\mathbf{Sym}_n^+$). This leads to a weak upper-semicontinuity result [1].

When it applies, C.I. yields dispersive (Strichartz-like) estimates. We thus learn that in Gas Dynamics, the internal energy cannot concentrate on zero-measure subsets. Other applications concern various models for particle dynamics: kinetic equations (Boltzman), mean-field models (Vlasov), molecular dynamics. The corresponding tensor is positive semi-definite whenever the particles interact pairwise according to a radial, repulsive force. Hard spheres dynamics shows that a Div-free tensor can be supported by a small set (here a graph), in which case a special form of C.I. is required [6].

Another relevant topic is that of multi-dimensional conservation laws, where it allows us to extend Kružkov's theory to L^p-data when p is finite, under a non-degeneracy assumption (collaboration with L. Silvestre [7]).

References

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