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# Global evolution of self-gravitating matter fields: nonlinear stability, asymptotics, and singularities

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I will present results on massive matter fields that evolve under their gravitational field. The initial value problem for the Einstein equations of general relativity, in a suitable gauge, can be formulated as a coupled system of nonlinear hyperbolic equations. Recent work by the author concerns the structure of these equations, the study of nonlinear competition taking place between the geometric/matter contributions, and new advances concerning solutions with low decay or with singularities. A general method of analysis of nonlinear wave equations and Klein-Gordon equations was introduced and applied to the Einstein equations. I will outline this method and indicate its potential in covering other models of nonlinear wave equations of interest. This is a joint work with Yue Ma (Xi'an Jiatong University).

Preliminary investigations of asymptotic flat spacetimes led physicists to suggest a *nonlinear instability mechanism* in the evolution of self-gravitating massive fields, even for arbitrarily small perturbations. Namely, the so-called family of "oscillating soliton stars" seemed to provide a potential candidate for instabilities that would develop during the evolution of massive matter governed by the Einstein equations. However, after several controversies the most recent numerical developments have led to the definite *conjecture* that, in asymptotically flat spacetimes, massive fields should be globally nonlinearly stable. Advanced numerical methods (including mesh refinement and high-order accuracy) were necessary and the references therein) and, in the long-time evolution of arbitrarily small perturbations of oscillating soliton stars, the following nonlinear mechanism was observed. In a first phase of the evolution, the matter tends to collapse and thus evolves toward the formation of a black hole. However, in an intermediate phase of the evolution, and below a certain threshold in the mass amplitude, the collapse phenomena significantly slows down. Eventually, the third and final phase of the evolution is reached and a strong dissipation mechanism dominates. It was thus conjectured by physicists that dispersion effects should be dominant in the long-time evolution of self-gravitating massive matter fields in the small perturbation and asymptotically flat regime. The work reported here provides a rigorous proof of this conjecture [5]. An independent proof was provided by Ionescu and Pausader [2].

## References

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