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Fast Three-Scale Singular Limits

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The equations treated in the theory of three-scale singular limits have the form

$$A^0(\varepsilon U)U_t + \sum_{j=1}^{d} A^j(U)U_{x_j} + \tfrac{1}{\delta}\mathcal{L}U + \tfrac{1}{\varepsilon}\mathcal{M}U = 0, \tag{1}$$

in which $\mathcal{L}$ and $\mathcal{M}$ are constant-coefficient antisymmetric operators and the two small parameters δ and ε tend to zero at different rates, specifically

$$\delta \to 0, \qquad \varepsilon \to 0, \qquad \mu := \tfrac{\delta}{\varepsilon} \to 0. \tag{2}$$

The original work of Cheng, Ju, and Schochet [1] on three-scale singular limits considered only the slow case, which for fixed initial data U_0 means that both $\mathcal{L}U_0$ and $\mathcal{M}U_0$ must vanish.

Two recent results will be presented in which this restriction on the initial data is lifted. The first result [3] is for systems having the simplified form

$$A^0(\varepsilon w(x))U_t + \sum_{j=1}^{d} A^j(U)U_{x_j} + \tfrac{1}{\delta}\mathcal{L}U = 0 \tag{3}$$

that retains the fundamental distinguishing feature of (1), namely that the small parameter δ whose inverse appears in the large term is much smaller than the small parameter ε appearing in the coefficient A^0 of the time derivative. Besides a uniform existence theorem, a novel convergence theorem is obtained for spatially-periodic solutions to (3), involving filtering by a variable-coefficient fast operator.

The second result [2] is for the stratification-dominated three-scale singular limit of the rotating stratified Boussinesq equations

$$\begin{aligned} &\frac{\partial}{\partial t}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} + \varepsilon^{-1}\mathbf{e}_3 \times \mathbf{v} = -\nabla\phi - \delta^{-1}\rho\mathbf{e}_3, \\ &\frac{\partial}{\partial t}\rho + (\mathbf{v}\cdot\nabla)\rho = \delta^{-1}w, \\ &\operatorname{div}\mathbf{v} = 0 \end{aligned} \tag{4}$$

in $\mathbb{R}^3$. The main novelty is a three-scale Strichartz estimate.

These results are joint work with Xin Xu and Pengcheng Mu, respectively.

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References

[1] B. Cheng, Q. Ju, and S. Schochet. Three-scale singular limits of evolutionary PDEs. *Arch. Ration. Mech. Anal.*, 229(2):601–625, 2018.

[2] Pengcheng Mu and Steve Schochet. Dispersive estimates for the inviscid rotating stratified Boussinesq equations in the stratification-dominant three-scale limit. *J. Math. Pures Appl. (9)*, 158:90–119, 2022.

[3] S. Schochet and X. Xu. Towards Uniform Existence and Convergence Theorems for Three-scale Systems of Hyperbolic PDEs with General Initial Data. preprint, 2022.