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High-order fully well-balanced methods for one-dimensional blood flow model with discontinuous mechanical and geometrical properties

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One-dimensional blood flow models have been extensively used to study wave propagation phenomena in arteries and veins. They are useful for the description of some aspects in the venous system such as the collapse of veins, which in some cases can be the responsible of some neurodegenerative diseases such as idiopathic parkinson or multiple sclerosis.

We are interested in the numerical study of the one-dimensional blood flow model with discontinuous mechanical and geometrical properties proposed in [1]. In [2] this model was revisited, completing the mathematical analysis. Together with this analytical study, well-balanced schemes based on the ones in [3] were designed. They were able to preserve only *blood-at-rest* stationary solutions.

Our purpose in this work is to develop high-order fully well-balanced numerical methods that are able to preserve every stationary solutions, not only the *blood-at-rest* ones. In order to do this we rewrite the system in a new compact form and we apply the well-balanced methodology developed in [4] based on the Generalized Hydrostatic Reconstruction [5]. we describe in detail the stationary solutions obtained from the system. Several tests are shown in order to compare the schemes in [2] and the new ones. We observe an improvement in the convergence to the exact solution, highlighting the importance of the fully well-balanced property.

References

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