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Comparison of divergence error and conventional energy error based p-adaptive discontinuous Galerkin methods in higher-order solutions of time-domain Maxwell's equations

Apurva TIWARI [*], Avijit CHATTERJEE [†]

It has been a constant endeavour to devise computational methods for higher-order accurate solutions to engineering problems with lower computational cost. One such means to achieve higher computational efficiency is the use of mesh adaptive methods. Classically the Finite Element Methods (FEM) in computational mechanics and more recently the popular Discontinuous Galerkin Finite Element Method (DGFEM) have spurred development of hp adaptive methods to make optimum use of the degrees of freedom employed to solve a given problem. Often driving these methods are features a solution must successfully resolve, based on specific physical or numerical phenomena. These are easy to implement but can require an expert user to suitably devise the same to deliver reasonable results [1]. A common criticism of feature driven methods is that they may not show direct relations with numerical errors that an adaptive method should eliminate. This leads to the more exacting error estimation based methods for hp adaptivity. The working principle behind these is that regions showing relatively large errors due to lack of requisite spatial accuracy should attract targeted computational effort in subsequent iterations. There is fair amount of research focussing on error estimation based methods [2, 3] and τ-estimation procedures [4] in adaptive FEM and DGFEM frameworks. Such developments found natural applications in structural mechanics, and a multitude of robust error indicators and estimators were developed to drive these methods. In this work, we explore extending this knowledge of error estimation and its principles, to drive adaptive algorithms in the linear wave dominated problems of Computational Electromagnetics (CEM). In this context we investigate the commonly electromagnetic (EM) energy driven p-adaptive methods for EM scattering problems in a Discontinuous Galerkin Time Domain (DGTD) framework. We study how well properties of established error estimators in computational mechanics are replicated for analogous EM energy based error indicators. Along with several practical traits that EM energy shares with strain energy in structural mechanics like being a scalar, co-ordinate system invariant and readily computable, it also shows a definite convergence with finer discretizations, similar to strain energy. We address shortcomings in making such a comparison from a theoretical standpoint, and show how a naive rephrasing of strain energy as error indicator in computational structural mechanics into EM energy in CEM does not serve the same functionality in driving higher-order accurate adaptive methods.

A recently proposed divergence error based driver [5] for p-adaptive DG solutions of the time-domain Maxwell's equations is contrasted with that based on EM energy. The divergence error based indicator builds on a re-interpretation of the numerical treatment of the Gauss constraints on divergence, in the time-domain Maxwell's equations. This numerical divergence error is usually disregarded as noise and may be safely ignored in practice. In [5], a shift in perspective is proposed by retaining these divergence errors, instead of eliminating them, as they can serve as error indicators for adaptive algorithms without affecting formal accuracy of the solution. In this work, we study the nature of error in EM energy as a driver motivated by conventional error estimators, and contrast it with a novel divergence error based driver. We show that a divergence error based indicator is theoretically superior to that based on EM energy error as it provides a better estimate for the relative truncation error besides sharing regular traits of being a scalar quantity which is easily computable. Numerical illustrations involving electromagnetic scattering from perfect conductors of various geometries, under transverse electric (TE) and transverse magnetic (TM) illumination are presented to demonstrate the two approaches.

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[*] Department of Aerospace Engineering, Indian Institute of Technology Bombay, India. Email: apurva.t@aero.iitb.ac.in
[†] Department of Aerospace Engineering, Indian Institute of Technology Bombay, India. Email: avijit@aero.iitb.ac.in

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