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Hysteretic Traffic Flow and Stop-and-go Waves

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Here, I report some of my recent works joint with Andrea Corli and Chi-Wang Shu.

Stop-and-go waves, also called phantom jams. occur frequently in vehicular traffic flows. Phantom jams can be generated by human driving behaviors alone, as demonstrated experimentally by [4]. They can also be triggered by lane changing or by road features such as bottlenecks, grade changes etc. Once formed, a phantom jam can persist for a long time.

Continuum traffic models of conservative form cannot produce stop-and-go patterns when both front of a stop-and-go wave are sharp, because one of the fronts violates the Lax shock condition.

Driver's hysteretic behavior were observed by Treiterer & Myers [5] in 1974. Figure 1 illustrates this behavior. In some traffic

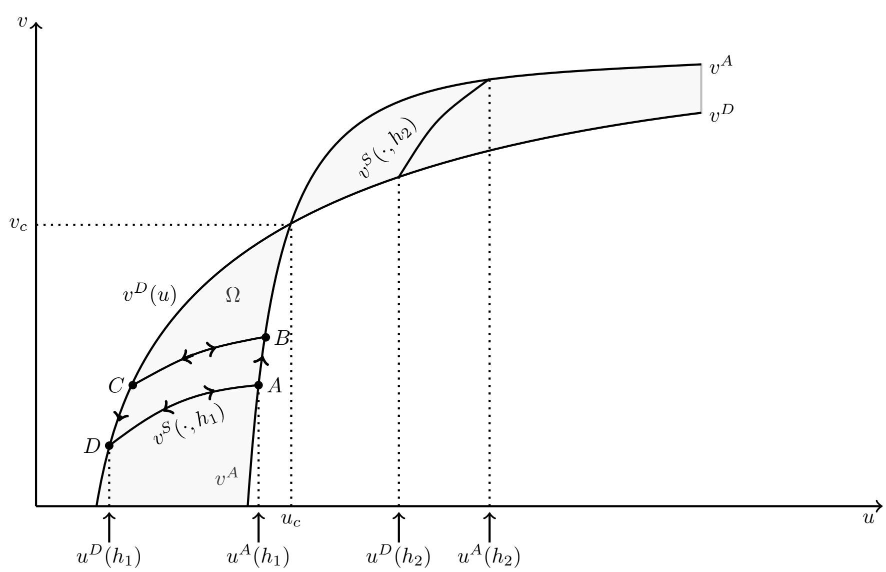

Figure 1: The speeds $v = v^A(u)$, $v = v^D(u)$ and $v = v^S(u,h)$ are for vehicles in acceleration, deceleration and scanning mode respectively. In acceleration mode, the vehicle's speed can increase along $v = v^A(u)$ only, such as from point A to B. Arrows indicate the direction the speed can change in each mode. At B, if the driver has to decelerate due to tightening of spacing u, he/she decelerates along the scanning curve BC, not along BA, due to either mental inertia or not wanting to loss momentum and hoping the spacing tightening is temperary. Between BC, the driver is said to be in scanning mode. If u keeps tightening, he/she will decelerate to point C. After that he/she will have to decelerate faster for safety, reaching D along the curve $v = v^D(u)$. In this case, the driver is said to be in deceleration mode. If u starts improving from there, then he/she will accelerate towards A. The loop ABCDA is called a hysteresis loop. The parameter h is to parametrize scanning curves $v = v^S(u,h)$, so $h = const$ along each scanning curve. The u-coordinate of the point of intersection of $v = v^D(u)$ (or $v = v^A(u)$) and $v = v^S(u,h)$ is denoted as $u^D(h)$ (or $u^A(h)$). The inverse of $u = u^A(h)$ (or $u = u^D(h)$) is denoted as $h = h^A(u)$ (or $h = h^D(u)$). An example of h parametrization is to assign $u^D(h) = h$ and hence $h^D(u) = u$.

literature, it is believed to cause phantom jam. However, there were no proof for such a conjecture, until [1]. It is known that the

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classical Lighthill-Whitham-Richards traffic flow model

$$u_t - v(u)_x = 0 \tag{1}$$

does not produce stop-and-go wave. Here, x is the Lagrange coordinate of vehicles, u the spacing, v the vehicle speed. We showed that by adding hysteresis effect in (1), stop-and-go waves emerge.

We incorporate hysteretsis in the LWR model to get the following model for hysteretic 1-lane traffic flow

$$\begin{cases} u_t - v(u,h)_x = 0, \\ h_t = \chi(A)h^A(u)_t + \chi(D)h^D(u)_t \end{cases} \tag{2}$$

where the boolean variables A, D, S are defined as

$$A = \left\{u = u^A(h),\ u_t > 0\right\}, \qquad D = \left\{u = u^D(h),\ u_t < 0\right\}, \tag{3}$$

$$S = \left\{u^A(h) \geq u \geq u^D(h),\ A = \mathit{false} \text{ and } D = \mathit{false}\right\}, \tag{4}$$

and $\chi()$ is the characteristic function.

In [1], it is shown that (2) has stop-and-go solutions shown in Figure 2

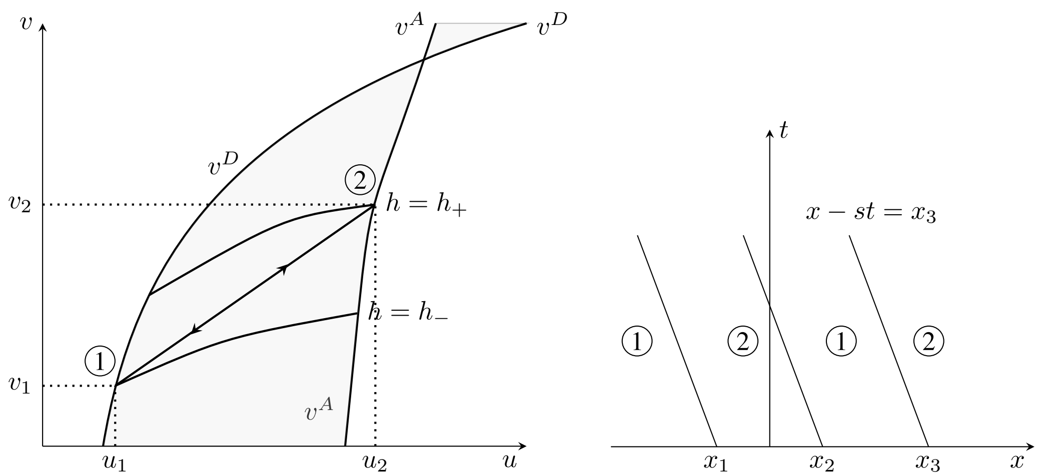

Figure 2: Stop-and-go solutions: left, in the (u, v)-plane; right, in the (x, t)-plane.

The meaning of weak solutions of (2) is defined in [2]. An first order upwinding approximation scheme for (2) is shown to be total variation diminishing in v, h, and hence they satisfy maximum and minimum principles. The limit of a convergent subsequence generated by the approximation scheme is shown to be a weak solution of the model if it is piecewise C^1.

In [3], we proposed a second order method and a limited variation WENO numerical scheme for the system (2). The limited variation WENO method is fifth order accurate in spatial coordinate and third order accurate in time steps. The proposed high resolution methods produce the desired solutions more efficiently than the first order upwinding method. These methods are total variation diminishing for the flux, and monotonicity-preserving for the flux and the hysteresis state variable.

References

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