A first-order hyperbolic reformulation of the Navier-Stokes-Korteweg system

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We present a novel first-order hyperbolic reformulation of the Navier-Stokes-Korteweg system, based on the Godunov-Peshkov-Romenski model of continuum mechanics[1], combined with an augmented Lagrangian approach[2], allowing to cast the non-linear dispersive Euler-Korteweg equations into a first-order hyperbolic system. The latter method is based on a classical penalty method used to approximate the gradient terms in the Lagrangian by a new set of independent variables, for which suitable closure equations are sought. The governing equations for the new introduced degrees of freedom admit curl-free constraints which must be taken into account in order to obtain stable numerical solutions. Thus, we employ here a thermodynamically compatible generalized Lagrangian multiplier (GLM) approach [3], similar to the GLM divergence cleaning introduced by Munz *et al.* [4] for the divergence constraint of the magnetic field in the Maxwell and MHD equations.

The system of equations is solved at the aid of a high-order ADER Discontinuous Galerkin finite-element scheme with *a posteriori* subcell finite volume limiting in order to deal with shock waves, discontinuities and steep gradients in the numerical solution. We show numerical results for several standard benchmark problems, including Ostwald ripening in one and two space dimensions, diffuse and dispersive traveling wave solutions.

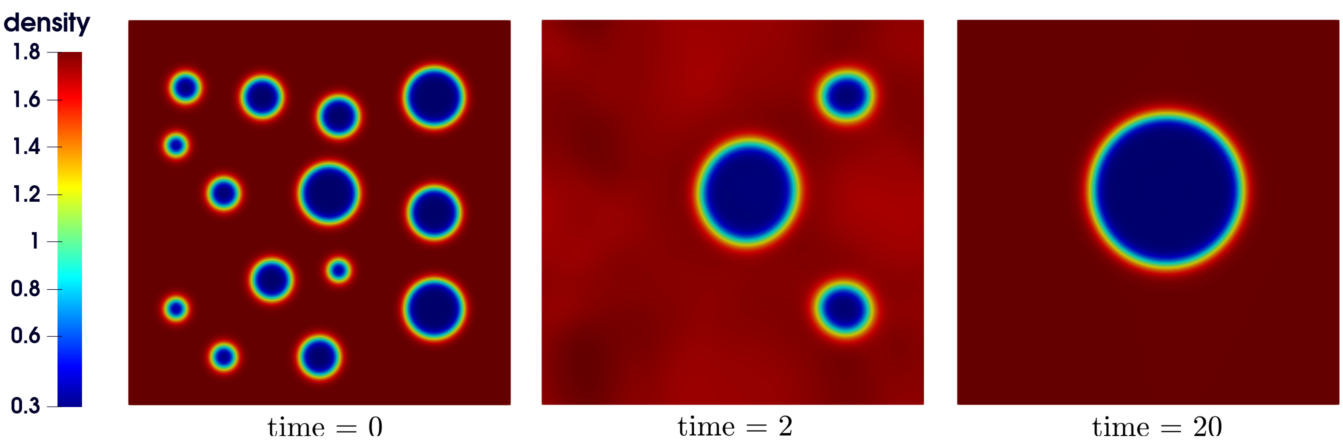

Figure 1: Ostwald ripening in two dimensions. The pictures represent the initial state, an intermediate state and the equilibrium state, respectively. The bubbles of lower size shrink and their masses are transferred to the bigger ones until one equilibrium bubble remains.

Acknowledgements

The authors acknowledge the financial support received from the Italian Ministry of Education, University and Research (MIUR) in the frame of the Departments of Excellence Initiative 2018–2022 attributed to DICAM of the University of Trento (grant L. 232/2016) and in the frame of the PRIN 2017 project *Innovative numerical methods for evolutionary partial differential equations and applications*. F.D. was also funded by a *UniTN starting grant* of the University of Trento.

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