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Analysis and numerics of the propagation speed for hyperbolic reaction-diffusion models

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We briefly discuss different models for describing reaction-diffusion phenomena based on hyperbolic equations. The standard approach makes use of parabolic systems which are, indeed, well suited to explain events such as heat transmission in close-to-equilibrium regimes. Nevertheless, such modeling is criticizable for several reasons, for instance the prediction of an infinite speed of propagation, the lack of time-delay and related inertial effects [3, 8], and the exceptionality of well-posed boundary value problems. In addition, in many contexts the hyperbolic corrections are relevant for applications: dynamics of biological tissues [2, 12, 13], population growth [10], forest fire models [11], etc... We adopt the point of view that a description by means of hyperbolic models – starting from the basic example of the *telegraph equation* – is viable, and more appropriate when the relaxation time required to perceive changes of the overall phenomenon is sufficiently large as compared to the diffusivity coefficient. As a matter of fact, differences may emerge in the transient regimes, whose cumulations may influence significantly the final outcome.

Actually, the emphasis is placed on the numerical computation of the propagation speed of special traveling wave solutions, namely propagating fronts, of hyperbolic dissipative processes. Three basic numerical schemes are presented, two of which can also be applied to general hyperbolic systems (with reduced performance when dealing with discontinuous initial data), and we compare their performance with respect to providing effective approximations of the propagation speed. Therefore, we focus on a specific class of 2x2 systems corresponding to second order PDEs in one space dimension, which are adapted for simplified modeling of reaction-diffusion equations with monostable [4, 1] and bistable reaction terms.
Beside the *phase-plane algorithm* which is convenient for approximating hyperbolic reaction-diffusion systems with damping, especially in cases with available explicit formulas [5, 6], we propose two PDE-based numerical schemes, the so-called *scout&spot algorithm* – based on tracking the level curve of some intermediate value of the wave profile – and the *LeVeque-Yee formula* [9, 7] – given by the average value of the discrete transport velocity – by assessing their capability in comparative experiments of genuine predictions.

We draw attention to the fact that we do not merely regard hyperbolic equations as perturbation of the limiting parabolic counterparts, and then the corresponding numerical schemes as a tool for approximating parabolic equations. But rather, we focus on hyperbolic models considered as an alternative language for describing dissipative mechanisms which are particularly interesting in far-from-equilibrium regimes.

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References

[1] E. Bouin, V. Calvez, G. Nadin. Hyperbolic traveling waves driven by growth. *Math. Models Meth. Appl. Sci.* 24(6): 1165–1195, 2014.

[2] S.R. Dunbar, H.G. Othmer. On a nonlinear hyperbolic equation describing transmission lines, cell movement, and branching random walks. Nonlinear oscillations in biology and chemistry (Salt Lake City – Utah, 1985) *Lecture Notes in Biomath.* 66: 274–289, Springer, Berlin, 1986.

[3] C. Eckart. The thermodynamics of irreversible processes, I. The simple fluid, II. Fluid mixtures, III. Relativistic theory of the simple fluid. *Phys. Rev.* 58: 267–269, 269–275, 919–924, 1940.

[4] K.P. Hadeler. Hyperbolic travelling fronts. *Proc. Edinb. Math. Soc.* 31(1): 89–97, 1988.

[5] C. Lattanzio, C. Mascia, R.G. Plaza, C. Simeoni. Analytical and numerical investigation of traveling waves for the Allen-Cahn model with relaxation. *Math. Models Methods Appl. Sci.* 26(5): 931–985, 2016. *– https://hal.archives-ouvertes.fr/hal-02525347*

[6] C. Lattanzio, C. Mascia, R.G. Plaza, C. Simeoni. Kinetic schemes for assessing stability of traveling fronts for the Allen-Cahn equation with relaxation. *Appl. Numer. Math.* 141: 234–247, 2019. *– https://hal.archives-ouvertes.fr/hal-02520750*

[7] C. Lattanzio, C. Mascia, R.G. Plaza, C. Simeoni. Analysis and numerics of the propagation speed for hyperbolic reaction-diffusion models. SEMA-SIMAI Springer Series, Partial Differential Equations: Ambitious Mathematics for Real-Life Applications, Donatella Donatelli and Chiara Simeoni (Editors), 2022 (in press)

[8] L. Lehner, O.A. Reula, M.E. Rubio. Hyperbolic theory of relativistic conformal dissipative fluids. *Phys. Rev. D* 97(2): 024013, 2018.

[9] R.J. LeVeque, H.C. Yee. A study of numerical methods for hyperbolic conservation laws with stiff source terms. *J. Comput. Phys.* 86(1): 187–210, 1990.

[10] V. Méndez, J. Camacho. Dynamics and thermodynamics of delayed population growth. *Phys. Rev. E Stat. phys. plasmas fluids relat. interdiscip. topics* 55(6): 6476–6482, 1997.

[11] V. Méndez, J.E. Llebot. Hyperbolic reaction-diffusion equations for a forest fire model. *Phys. Rev. E Stat. phys. plasmas fluids relat. interdiscip. topics* 56(6): 6557–6563, 1997.

[12] H.G. Othmer, S.R. Dunbar, W. Alt. Models of dispersal in biological systems. *J. Math. Biol.* 26(3): 263–298, 1988.

[13] F. Xu, K.A. Seffen, T.J. Lu. Non-Fourier analysis of skin bio-thermomechanics. *Int. J. Heat Mass Transf.* 51(9-10): 2237–2259, 2008.