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A filtering framework for Finite Volume/Element Schemes

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The Smoothness-Increasing Accuracy-Conserving (SIAC) filter is a post-processor designed to extract accuracy from DG and FEM solutions exploiting the natural superconvergence of these methods. The filter can be used to capture shocks as well as for feature detection during fluid flow visualization: it reduces oscillations in the errors and increases convergence rates, leading in general, to lower errors. Although the filtering operation has a simple mathematical formulation, designing and implementing an effective tool for general applications implies dealing with domain boundaries, nonuniform meshes and computational efficiency. This work presents a numerical framework and a software package of various SIAC filters with applications to hyperbolic equations and multi-resolution analysis.

Introduction

Numerical filtering can be exploited during simulations to detect discontinuities as well as for feature detection during fluid flow visualization. Typically, the filtering operation removes noise from signals and/or unwanted scales in a multi-scale problem via convolution:

$$u^\star(x) = \int_{\mathbb{R}} K_H(y-x)u(y)dy.$$

Here, u is a quantity of interest and K_H a scaled kernel function, carefully designed to modify the behavior of u in an application optimal way. The Smoothness-Increasing Accuracy-Conserving (SIAC) filter exploits the idea of superconvergence in the underlying numerical method. It reduces oscillations in the errors and generally, the error magnitude as well. In Figure 1 we show the traditional SIAC kernel: a linear combination of central B-spline functions.

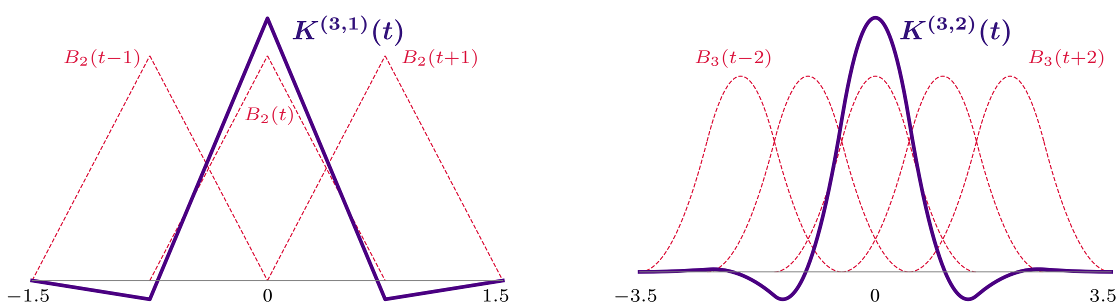

Figure 1: Two kernels $K^{(\cdot,\cdot)}$ built with three and five B-splines of order 2 (left) and 3 (right), respectively.

A standalone tool

While the concept is straightforward, designing and implementing an effective filter for general applications implies dealing with domain boundaries, filter scaling choices for nonuniform meshes, and computational efficiency. We have created an open-source software tool with the following structure: the user supplies two files: one containing the mesh information (GMSH format) and

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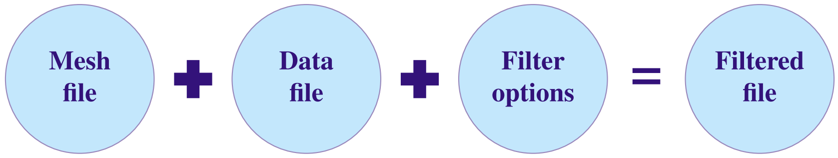
another containing the corresponding approximation data sampled at each element. Then, the user prescribes filtering parameters such as the desired level of accuracy and smoothness. The filtered output at the sampled points is then returned.



Applications

In Figure 2 we show a simple two-dimensional example of how the superconvergent Line SIAC filter [2] can reduce both the numerical errors and oscillations. The SIAC filters have been applied to Aeroacoustic problems, visualization of streamlines and isosurfaces, adaptivity and shock regularization. Recently, it is used for constructing more accurate multi-resolution simulations that allows for moving coarse data to finer grids [3].

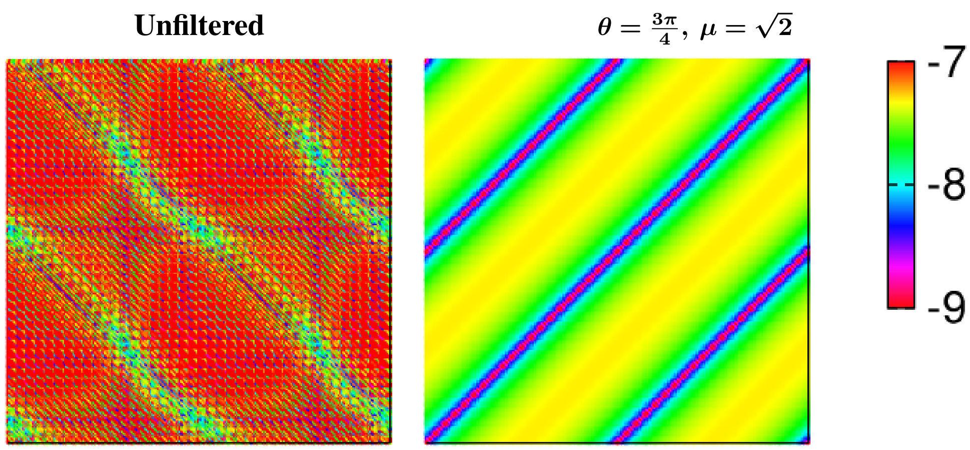


Figure 2: Contour line error plots (log) before and after filtering a fourth order DG approximation to a linear advection equation with $u_0(x, y) = \sin x \cdot \cos y$ over a $N = 40 \times 40$ uniform mesh applying a Line SIAC Filter.

References

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[3] M.J. Picklo and J.K. Ryan. *Enhanced Multi-Resolution Analysis for Multi-Dimensional Data Utilizing Line Filtering Techniques.* arXiv 2108.05769 (2021).