XVIII International Conference on Hyperbolic Problems
Theory, Numerics, Applications
Málaga, June 20-24, 2022

High order well balanced schemes for shallow water equations in covariant coordinates

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In this work we present a family of high order accurate well balanced (WB) Finite Volume (FV) and Discontinuous Galerkin (DG) schemes for the solution of a novel formulation of the shallow water (SW) equations written through a Local Covariant System (LCS) of variables.

The employed differential problem is derived by integrating the incompressible Navier-Stokes (NS) equations along paths that are *everywhere orthogonal* to the bathymetry, following [1]. This is in contrast with the classical derivation of the SW equations, which generally, despite the geometry of the bathymetry, are obtained by integrating NS along directions parallel to the gravity force. However, this classical method of integration is not exact in the case of curved bottoms. In particular, non-negligible differences in peak values or in the shape of hydrographs are evident when comparing numerical solutions obtained with the proposed Local Covariant approach with the models derived regardless of the bathymetry during the NS integration process [1].

The proposed model in covariant variables allows both to consider particular geometries at the bottom of the surface and at the same time to satisfy the shallow water hypothesis along the cross-flow surfaces, i.e. the surfaces normal to the fluid velocity field. Moreover, the two special cases of SW equations for flat bathymetry in general Cartesian coordinates, and spherical bathymetry in spherical coordinates with NS equations integrated along the radial directions [4, 5], can be reproduced by our model.

To solve the system numerically, we employ both a simple and robust Finite Volume scheme of order two, and Discontinous Galerkin schemes, of arbitrary high order of accuracy both in space and in time, based on the fully discrete one-step ADER paradigm [2]. Furthermore, we equip these high order accurate schemes with simple but effective well balanced techniques [3], which, by exploiting the knowledge of the steady state solutions of interest, allow to obtain numerical methods which are *exact*, up to machine precision, on equilibrium solutions and more accurate that the underlying schemes when studying small perturbations arising around the equilibria.

We close the presentation by showing the obtained numerical results and an overview on future research directions.

Acknowledgements

This work has been supported by the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement SuPerMan (No 101025563).

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