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Geometric optics for quasilinear hyperbolic boundary value problems

Corentin Kilque *

We take interest into solutions to hyperbolic quasilinear boundary value problems with highly oscillating forcing boundary term. The aim is to construct an approximation, in the high frequency asymptotic, of the exact solution to this problem, in the form of a WKB expansion. In [7], [1], and [2], the authors study the same problem but with only one phase on the boundary. Here we consider multiple phases on the boundary: this is the multiphase framework. Because of the nonlinearity, in the approximate solution, a countable infinite number of linear phases are generated inside the domain. We shall therefore take interest into the functional framework adapted to the study of this problem, namely of almost periodic functions.

This functional framework has been previously used for constructing approximate solutions to semilinear problems, in the framework of Wiener algebra, notably by [3] for the Cauchy problem and [6] for the boundary value problem, as well as for quasilinear problems (in the framework of Bohr almost periodic functions), especially by [4] for the Cauchy problems. It would be discussed in this contribution a similar result than [4] for a quasilinear boundary value problem, see [5], namely existence and uniqueness for the leading term of the geometric optics asymptotic expansion. This leading profile is obtained as solution to a quasilinear problem taking into account the potentially infinite number of resonances between phases. This quasilinear problem is solved by deriving a priori estimates without loss of derivatives.

References

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*Institut de Mathématiques de Toulouse - 118, route de Narbonne - F-31062 Toulouse Cedex 9. Email: corentin.kilque@math.univ-toulouse.fr