**XVIII International Conference on Hyperbolic Problems**
**Theory, Numerics, Applications**
**Málaga, June 20-24, 2022**

# Qualitative Properties of a Mathematical Model For Data Flow

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Current and future generations of extreme scale computers have achieved and are expected to achieve increases in performance via greater levels of parallelism at multiple levels, e.g. within the processors as well as increasing the number of processors and nodes. Therefore, a need for predictive, quantitative models of computational performance is growing.

Recently, in [1] a deterministic microscopic model of data processing and flow in an extreme scale computer with interprocessor communications and asynchronous executions has been proposed. Then, the fluid-limit model which treats such a machine in a continuum framework is derived in order to have potential benefits, such as the simulation of data flow in large systems and the possibility of using the theoretical tools of partial differential equations to study the performance of these systems.

In this talk, we reformulate the macroscopic model of data flow as a conditioned hyperbolic system of partial differential equations and analyze weak solutions. Further, we present the relaxation system associated to the model in order to propose an implicit-explicit discretization of the system and, consequently, to numerically investigate the solutions of the model. Finally, we analyze a control problem where the objective is the minimization of the processing time. The optimal control is then applied on the distribution of the processing rate among the processors.

## Acknowledgements

MH and GV thank the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) for the financial support through 20021702/ GRK2326, 333849990/IRTG-2379, HE5386/14,15,18-1,19-1,22-1 and under Germany?s Excellence Strategy EXC-2023 Internet of Production 390621612. The funding through HIDSS-004 is acknowledged. This work has been supported also by the US National Science Foundation, RNMS (KI-Net) grant 11-07444, and the U.S. Department of Energy, Office of Advanced Scientific Computing Research. The work of CDH was performed at the Oak Ridge National Laboratory, which is managed by UT-Battelle, LLC under Contract No. De-AC05-00OR22725.

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