XVIII International Conference on Hyperbolic Problems
Theory, Numerics, Applications
Málaga, June 20-24, 2022

Stabilizing high order shallow water solvers via parachutes

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We present a high-order accurate, positivity preserving and well-balanced finite volume scheme for the shallow water equations with variable topography. An unlimited third order scheme is combined with the recent, second-order accurate Bottom-Surface-Gradient Method (BSGM, [3]). This is monitored by an a-posteriori MOOD (Multidimensional Optimal Order Detection) limiting step [4, 5, 1], which detects possible local instabilities of a high-order candidate solution such as local oscillations or loss of positivity, and switches locally to a lower order, stable and robust "parachute" scheme if necessary.

The aim of this contribution is to clarify if a second-order accurate scheme can be robust enough to serve as a parachute. Our approach is closely related to [6, 7], who use a first-order scheme as parachute. Instead, we choose the BSGM, which is positivity-preserving, well-balanced and avoids non-physical reflections from bottom reconstructions, and is more accurate than a first-order scheme. If this stabilization should not be sufficient, we will use a first-order reserve-parachute such as [2]. Note that more generally, a "cascade" of parachutes was introduced in [1].

We will discuss the accuracy, effectiveness and robustness of the proposed adaptive methodology, both for near-equilibrium and non-equilibrium depth-averaged flows in one- and two-space dimensions.

Acknowledgements

This research has been partially supported by DFG grant GRK2326 (EDDy).

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