A quick review on Carlos Pares' main contributions on the numerical analysis of PDEs

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The objective of this talk is to give a quick overview of the most relevant contributions of Prof. Carlos Parés on the numerical analysis of PDEs. We will begin with one of his most cited early works on the analysis of the Stokes problem with non-standard boundary conditions and its extension to the Navier-Stokes equations and some applications (see [13, 26]). Next, we will present his main contributions on the approximation of variational inequalities and, in particular, of the study of the convergence of the Bermúdez-Moreno Algorithm and some applications (see [21, 22, 18]). Finally, we will present some of his most well-known contributions on the numerical approximation of hyperbolic systems with source terms and non-conservative products, such us the notion of path-conservative schemes ([17, 4, 8, 10]), the problem of the convergence towards the correct weak solutions ([6, 20, 9, 2, 24]) and the derivation of well-balanced schemes ([7, 11, 23, 12, 16]) and their applications ([3, 5, 14, 15, 19, 25]).

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