Layer-averaged models for complex rheologies. Well-balanced approximations

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In this talk first we present layer-averaged approximations, also named multilayer models (see [1]-[3]), to simulate granular avalanches with a $\mu(I)$-rheology and for Herschel-Bulkley model. These two rheologies have some similarities and some hugh differences. For example, in both cases we can observe in the simulations and in experimental measurements an important vertical structure of the velocity profile. In Figure 1 and 2 the vertical structure in terms of horizontal velocity profile is presented. The main difference is that we can observe very different profiles of the velocity. In granular avalanches the particles near the free surge present a great movility and we observe zero velocity near the bottom. Contrariously, in Hersche-Bulkley models we can observe a bigger gradient of the velocity near the bottom and a plug or pseudo-plug area, moving at constant velocity but not zero, near the free surface. This implies that both models have different forms of the stationary solutions. Thus, different strategies are proposed to obtain well-balanced finite volume approximations.

For the case of $\mu(I)$-rheology, several depth-averaged and laver-averaged models can be found in the bibilography that approximates Navier-Stokes system by asymptotic approximations (see [2, 4, 7]), but in any case all components of the stress tensor are considered, even in the case of weakly non-hydrostatic models. In [5] a layer-averaged model to approxiamte Herschel-Bulkley model is proposed, in this case by considering a hydrostatic pressure and the main order of the stress tensor component, by introducing and assymptotic analysis.

To finish this talk we will present the results introduced in [6], where a layer-averaged approximation of Navier-Stokes system with complex rheologies, taking into account all components of the stress tensor, is introduced.

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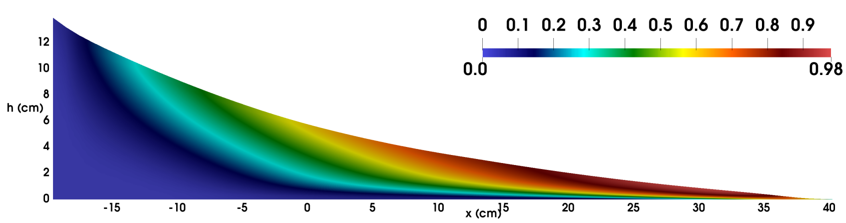

Figure 1: Dry granular avalanche with a $\mu(I)$-rheology

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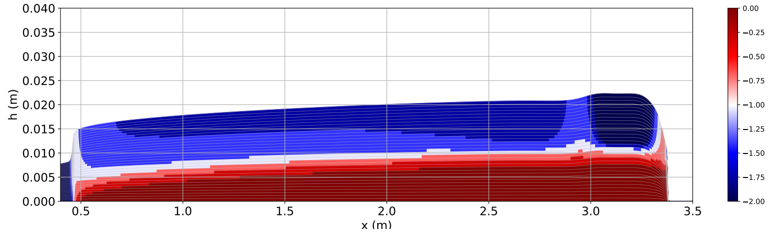

Figure 2: Avalanche with a Herschel-Bulkley model

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