Implicit-explicit numerical methods for the isothermal compressible Cahn-Hilliard-Navier-Stokes equations

Pep MULET*

In [4] a spinodal decomposition, governed by the Cahn-Hilliard equation [2], is conjectured as the underlying mechanism that explains the layered sedimentations of monodisperse colloidal particles.

Since the Cahn-Hilliard equation cannot explain this phenomenon by itself, the gravitational force is introduced into the model by means of conservation of mass and momentum, which, together with conservation of individual species and ignoring temperature changes, yields a system of equation, the isothermal Navier-Stokes-Cahn-Hilliard equations [3, 1], which are a system of fourth-order partial differential equations that model the evolution of mixtures of binary fluids under gravitational effects.

Although incompressible models for these equations might be more suitable for explaining the cited layering phenomenon, we consider the compressible case for the evolution of, e.g. foams, solidification processes, fluid-gas interface.

The aim of this work is to design implicit-explicit time-stepping schemes to avoid the severe restriction posed by the high order terms for the efficient numerical solution of boundary-initial problems with these equations.

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^{*}Dpt. Mathematics, Unversity of Valencia, Av. Vicent Andrés Estellés, 46100 Burjassot, Spain. Email: mulet@uv.es