WORKSHOP ON NUMERICAL APPROXIMATION OF HYPERBOLIC PDES IN HONOR OF PROF. CARLOS PARÉS' 60TH BIRTHDAY
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Selected topics on nonlinear waves with periodic structure

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Waves in periodic structrures have attracted a lot of attention in recent years. Here we consider three different problems. The first one concerns the development of efficient methods for the numerical treament of stratified fluids composed by a sequence of alternate layers. Such media show interesting macroscopic properties, which may be quite different from those of the individual fluids which constitute the stratified system. On a macroscopic scale, such a system can be considered a sort of *fluid metamaterial*. Numerical simulation of stratified multifluid may be challenging if one of the two fluids is much stiffer than the other, as is the case, for example, of air and water multilayer. For this reason, a novel method has been developed for the simulation of layered multifluid in Lagrangian coordinates [1]. The method exploits the special structure of the Lagrangian formulation of the Euler equations and is based on a simple and effective non conservative implicit predictor and a conservative corrector, which allows a very robust and efficient solution of the Euler equations, with no CFL stability restrictions and no need of complicated Riemann solvers. The method is second order accurate in space. High order accuracy in time is achieved by singly diagonally implicit Runge-Kutta schemes in time. Several numerical tests illustrate the efficiency and robustness of the method.

The other two topics concern problems arising in shallow water waves. In the first one we consider the Exner model of sedimentation, in which we study the behavior of wave train produced by a periodic signal at the inflow boudnary [2]. For long time simulation, the use of absorbing boundary conditions at the outflow boundary is essential to prevent formation of sourious solutions.

The second problem concerns the study of shallow water waves on a periodic bathymetry [3]. We consider the propagation of waves whose wavelength is much larger than the period of the bathymetry. After a long time, an initial Gaussian pulse splits into several waves of various amplitude. The phenomenon is explained in terms of dispersive waves satisfying a model system obtained from the original one by suitable asymptotic expansion. The solution to the model system is in good agreemet with the detailed numerical solution of the full SW system, with an agreement that improves with the number of terms in the expansion.

References

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